Grade Level/Course: Algebra 1
Lesson/Unit Plan Name: Solving Radical Equations
Rationale/Lesson Abstract: To provide students the best possible methods for solving equations that contains a radical.
Timeframe: 60 minutes for solving an equation with only one radical, 100 minutes for solving an equation with more than one radical.
Common Core Standard(s): • A-REI-2: Solve simple rational and radical equations in one variable and give examples how extraneous solutions may arise.

Instructional Resources/Materials:
One set of activity cards per pair of students. See activity at the end of lesson.

Lesson:

"These are some examples and non-examples of radical equations. Talk with your elbow partner and come up with a sentence that defines a radical equation." [*Have students share out before giving formal definition*]

Examples of Radical Equations	Non-Examples of Radical Equations
$\sqrt{x} + 5 = 11$	$\sqrt{5} + x^2 = 11$
$\sqrt[3]{x-4} = 7$	$x - 4 = \sqrt[4]{16}$
$4\sqrt{x-7} + 12 = 28$	$x\sqrt{10-7} + 12 = 28$
$\sqrt[5]{x} = 225$	$x^4 = \sqrt[3]{27}$

Definition of a Radical Equation: An equation where the variable is found underneath a square root, cube root or a higher root.

Definition of a Radicand: The number or expression under a radical symbol.

Ask students:

"What do we know about solving equations?" [*Isolate the variable*]

"Right, we do need to isolate the variable. What are some methods for isolating the variable?" [*Decompose, inverse operations, bar model*].

"Let's look at an equation with a variable as a radicand."

$$\sqrt{x} = 6$$

"For what value of *x* would you substitute in to make this equation a true statement?" [*36*]

"36, right, the $\sqrt{36} = 6$. Let's see all the algebra we could use to get to that answer."

$\sqrt{x} = 6$ $\sqrt{x} = \sqrt{36}$ $x = 36$	$\sqrt{x} = 6$ $\left(\sqrt{x}\right)^2 = 6^2$ $x = 36$	$\sqrt{x} = 6$ $x^{\frac{1}{2}} = 6$ $\left(x^{\frac{1}{2}}\right)^2 = \left(6\right)^2$ $x = 36$

"Let's solve some equations!"

Ex. 1 $\sqrt{x}+5=11$

$\sqrt{x}+5=11$ $\sqrt{x}+5=6+5$ $\sqrt{x}+\cancel{5}=6+\cancel{5}$ $\sqrt{x}=6$ $\sqrt{x}=\sqrt{36}$ $x=36$ **Check!**	$\sqrt{x}+5=11$ $\sqrt{x}+5-5=11-5$ $\sqrt{x}=6$ $\left(\sqrt{x}\right)^2=\left(6\right)^2$ $x=36$ **Check!**	$\sqrt{x}+5=11$ $\sqrt{x}+5-5=11-5$ $\sqrt{x}=6$ $x^{\frac{1}{2}}=6$ $\left(x^{\frac{1}{2}}\right)^2=\left(6\right)^2$ $x=36$ **Check!**

You Try: $\sqrt{x}-4=7$

$\sqrt{x}-4=7$ $\sqrt{x}-4=7+4-4$ $\sqrt{x}-\cancel{4}=7+4-\cancel{4}$ $\sqrt{x}=11$ $\sqrt{x}=\sqrt{121}$ $x=121$ **Check!**	$\sqrt{x}-4=7$ $\sqrt{x}-4+4=7+4$ $\sqrt{x}=11$ $\left(\sqrt{x}\right)^2=\left(11\right)^2$ $x=121$ **Check!**	$\sqrt{x}-4=7$ $\sqrt{x}-4+4=7+4$ $\sqrt{x}=11$ $x^{\frac{1}{2}}=11$ $\left(x^{\frac{1}{2}}\right)^2=\left(11\right)^2$ $x=121$ **Check!**

"What is the difference between $\sqrt{x}-4=7$ and $\sqrt{x-4}=7$? "[The first has x as the radicand and -4 outside the square root and the other has the quantity $x-4$ as the radicand.]

"Let's solve $\sqrt{x-4}=7$ and compare the answer to $\sqrt{x}-4=7$"

Ex 2: $\sqrt{x-4}=7$

$\sqrt{x-4}=7$ $\sqrt{x-4}=\sqrt{49}$ $x-4=49$ $x-4=49+4-4$ $x-\cancel{4}=49+4-\cancel{4}$ $x=53$ **Check!**	$\sqrt{x-4}=7$ $\left(\sqrt{x-4}\right)^2=\left(7\right)^2$ $x-4=49$ $x-4+4=49+4$ $x=53$ **Check!**	$\sqrt{x-4}=7$ $\left[\left(x-4\right)^{\frac{1}{2}}\right]^2=\left(7\right)^2$ $x-4=49$ $x-4+4=49+4$ $x=53$ **Check!**

You Try: $\sqrt{x+6} = 15$

$\sqrt{x+6} = 15$ $\sqrt{x+6} = \sqrt{225}$ $x+6 = 225$ $x+6 = 219+6$ $x+\cancel{6} = 219+\cancel{6}$ $x = 219$ **Check!**	$\sqrt{x+6} = 15$ $\left(\sqrt{x+6}\right)^2 = \left(15\right)^2$ $x+6 = 225$ $x+6-6 = 225-6$ $x = 219$ **Check!**	$\sqrt{x+6} = 15$ $\left[\left(x+6\right)^{\frac{1}{2}}\right]^2 = \left(15\right)^2$ $x+6 = 225$ $x+6-6 = 225-6$ $x = 219$ **Check!**

Ex 3: $4\sqrt{x-7}+12 = 28$

"What terms make up the radicand?" [$x-7$]

"So, what do we need to isolate?" [$\sqrt{x-7}$]

$4\sqrt{x-7}+12 = 28$ $4\sqrt{x-7}+12 = 16+12$ $4\sqrt{x-7}+\cancel{12} = 16+\cancel{12}$ $4\sqrt{x-7} = 16$ $4\cdot\sqrt{x-7} = 4\cdot 4$ $\cancel{4}\cdot\sqrt{x-7} = 4\cdot\cancel{4}$ $\sqrt{x-7} = 4$ $\sqrt{x-7} = \sqrt{16}$ $x-7 = 16$ $x-7 = 16+7-7$ $x-\cancel{7} = 16+7-\cancel{7}$ $x = 23$ **Check!**	$4\sqrt{x-7}+12 = 28$ $4\sqrt{x-7}+12-12 = 28-12$ $4\sqrt{x-7} = 16$ $\dfrac{\cancel{4}\sqrt{x-7}}{\cancel{4}} = \dfrac{16}{4}$ $\sqrt{x-7} = 4$ $\left(\sqrt{x-7}\right)^2 = \left(4\right)^2$ $x-7 = 16$ $x-7+7 = 16+7$ $x = 23$ **Check!**	$4\sqrt{x-7}+12 = 28$ $\dfrac{4\sqrt{x-7}}{4}+\dfrac{12}{4} = \dfrac{28}{4}$ $\sqrt{x-7}+3 = 7$ $\sqrt{x-7}+3-3 = 7-3$ $\sqrt{x-7} = 4$ $\left(x-7\right)^{\frac{1}{2}} = 4$ $\left(\left(x-7\right)^{\frac{1}{2}}\right)^2 = 4^2$ $x-7 = 16$ $x-7+7 = 16+7$ $x = 23$ **Check!**

You Try: (use any method) $5\sqrt{x+3}-10=15$

$5\sqrt{x+3}-10=15$	$5\sqrt{x+3}-10=15$	$5\sqrt{x+3}-10=15$
$5\sqrt{x+3}-10=15-10+10$	$5\sqrt{x+3}-10+10=15+10$	$\dfrac{5\sqrt{x+3}}{5}-\dfrac{10}{5}=\dfrac{15}{5}$
$5\sqrt{x+3}-\cancel{10}=15-\cancel{10}+10$	$5\sqrt{x+3}=25$	$\sqrt{x+3}-2=3$
$5\sqrt{x+3}=25$	$\dfrac{5\sqrt{x+3}}{5}=\dfrac{25}{5}$	$\sqrt{x+3}-2+2=3+2$
$5\cdot\sqrt{x+3}=5\cdot5$	$\sqrt{x+3}=5$	$\sqrt{x+3}=5$
$\cancel{5}\cdot\sqrt{x+3}=5\cdot\cancel{5}$	$\left(\sqrt{x+3}\right)^2=\left(5\right)^2$	$(x+3)^{\frac{1}{2}}=5$
$\sqrt{x+3}=5$	$x+3=25$	$\left((x+3)^{\frac{1}{2}}\right)^2=\left(5\right)^2$
$\sqrt{x+3}=\sqrt{25}$	$x+3-3=25-3$	
$x+3=25$	$x=22$	$x+3=25$
$x+3=22+3$		$x+3-3=25-3$
$x+\cancel{3}=22+\cancel{3}$	**Check!**	$x=22$
$x=22$		
Check!		**Check!**

"What if our equation was $\sqrt{x}=-5$? Can you think of a number that when you take the square root gives you -5?" [25? No, -25! There isn't one]

"Right, there isn't a number in the Real Number Set that when you take the square root gives you -5 as an answer. In this case we would write 'no real solution'."

"With your partner, describe and correct the error you see in this problem. Check the answer shown to justify your response."

$$\sqrt{3x}+9=0$$
$$\sqrt{3x}+9-9=0-9$$
$$\sqrt{3x}=-9$$
$$\left(\sqrt{3x}\right)^2=\left(-9\right)^2$$
$$3x=81$$
$$3\cdot x=3\cdot27$$
$$x=27$$

[The problem is $\sqrt{3x}=-9$. At this point there is no solution in the real numbers.]

More Than One Radical

"Sometimes we will encounter a problem with more than one radical. What do you think we should do here? Same idea applies, we will square (cube, raise to the power needed) both sides to eliminate the radical."

Ex 4: $\sqrt{x-3} = \sqrt{2x+4}$

You Try:

$\sqrt{3x+5} = \sqrt{x-6}$

(Use any method)

$\sqrt{x-3} = \sqrt{2x+4}$ $x-3 = 2x+4$ $x-3 = x+x+4$ $\cancel{x}-3 = \cancel{x}+x+4$ $-3 = x+4$ $-3+4-4 = x+4$ $-3+\cancel{4}-4 = x\cancel{+4}$ $-7 = x$ $x = -7$ **Check!**	$\sqrt{x-3} = \sqrt{2x+4}$ $\left(\sqrt{x-3}\right)^2 = \left(\sqrt{2x+4}\right)^2$ $x-3 = 2x+4$ $x-x-3 = 2x-x+4$ $-3 = x+4$ $-3-4 = x+4-4$ $-7 = x$ $x = -7$ **Check!**	$\sqrt{x-3} = \sqrt{2x+4}$ $\left[(x-3)^{\frac{1}{2}}\right]^2 = \left[(2x+4)^{\frac{1}{2}}\right]^2$ $x-3 = 2x+4$ $x-3 = x+x+4$ $-3 = x+4$ $-3-4 = x+4-4$ $-7 = x$ $x = -7$ **Check!**	$\sqrt{3x+5} = \sqrt{x-6}$ $\left(\sqrt{3x+5}\right)^2 = \left(\sqrt{x-6}\right)^2$ $3x+5 = x-6$ $3x-x+5 = x-x-6$ $2x+5 = -6$ $2x = -11$ $\dfrac{2x}{2} = \dfrac{-11}{2}$ $x = \dfrac{-11}{2}$ **Check!**

Ex. 5: $\sqrt{x-1} = x-7$

$\sqrt{x-1} = x-7$ $\left(\sqrt{x-1}\right)^2 = (x-7)^2$ $x-1 = (x-7)(x-7)$ $x-1 = x^2-14x+49$ $x-x-1+1 = x^2-14x-x+49+1$ $0 = x^2-15x+50$ $0 = (x-5)(x-10)$ $\therefore x = 5 \text{ and } 10$	**Extraneous Solutions**: Solutions that do not satisfy the original equation. Must check each solution in the original equation.

Let $x = 5$	Let $x = 10$
$\sqrt{x-1} = x-7$ $\sqrt{5-1} = 5-7$ $\sqrt{4} \neq -2$	$\sqrt{x-1} = x-7$ $\sqrt{10-1} = 10-7$ $\sqrt{9} = 3$

You Try: $\sqrt{x-2} = -x+4$

$\sqrt{x-2} = -x+4$

$\left(\sqrt{x-2}\right)^2 = \left(-x+4\right)^2$

$x-2 = \left(-x+4\right)\left(-x+4\right)$

$x-2 = x^2 - 8x + 16$

$0 = x^2 - 9x + 18$

$0 = \left(x-3\right)\left(x-6\right)$

$x = 3$ and 6

Check for Extraneous Solutions

Let $x = 3$	Let $x = 6$
$\sqrt{x-2} = -x+4$	$\sqrt{x-2} = -x+4$
$\sqrt{3-2} = -3+4$	$\sqrt{6-2} = -6+4$
$\sqrt{1} = 1$	$\sqrt{4} \neq -2$

Activity:

Directions: Put students into groups of 2-4. Cut out pieces (steps and reasons are not in order except for the given equation and check). Students will put the solution steps in order as well as matching the reason to each step. Have them copy examples into their notebook (or glue pieces in their notebook).

Proof 1

Steps	Reasons
$\sqrt{2x-1}+5=8$	Given Equation
$2x-1=9$	Simplify
$\sqrt{2x-1}=3$	Square both sides to eliminate the square root.
$2x=10$	Division Property of Equality
$\sqrt{2x-1}+5-5=8-5$	Isolated square root
$\left(\sqrt{2x-1}\right)^2=(3)^2$	Addition Property of Equality
$x=5$	Subtraction Property of Equality

$\dfrac{2x}{2} = \dfrac{10}{2}$	Simplify
$2x - 1 + 1 = 9 + 1$	Simplify
$\sqrt{2(5) - 1} + 5 = 8$ $\sqrt{10 - 1} + 5 = 8$ $\sqrt{9} + 5 = 8$ $3 + 5 = 8$	Check

Proof 2

Steps	Reasons
$x - 1 = \sqrt{5x - 9}$	Given Equation
$x^2 - 2x + 1 = 5x - 9$	Binomial Expansion
$(x - 1)(x - 1) = 5x - 9$	Factored

$\left(x-1\right)^{2}=\left(\sqrt{5x-9}\right)^{2}$	Used Zero Product Property
$x = 5$ or 3	Square both sides to eliminate the square root.
$\left(x-5\right)\left(x-3\right)=0$	Set equal to zero
$x^{2}-7x+10=0$	Multiplied Binomials
$5-1=\sqrt{5(5)-9}$ $\quad$ $3-1=\sqrt{5(3)-9}$ $4=\sqrt{16}$ $\qquad\quad$ $2=\sqrt{4}$ $2\neq\sqrt{6}$ $\qquad\quad$ $2=2$	Check for Extraneous Solutions!

Answer Key:

Steps	Reasons
$\sqrt{2x-1}+5=8$	Given Equation
$\sqrt{2x-1}+5-5=8-5$	Subtraction Property of Equality
$\sqrt{2x-1}=3$	Isolate square root
$\left(\sqrt{2x-1}\right)^2=\left(3\right)^2$	Square both sides to eliminate the square root.
$2x-1=9$	Simplify
$2x-1+1=9+1$	Addition Property of Equality
$2x=10$	Simplify
$\dfrac{2x}{2}=\dfrac{10}{2}$	Division Property of Equality
$x=5$	Simplify
$\sqrt{2(5)-1}+5=8$ $\sqrt{10-1}+5=8$ $\sqrt{9}+5=8$ $3+5=8$	Check

Steps	Reasons
$x-1=\sqrt{5x-9}$	Given Equation
$\left(x-1\right)^2=\left(\sqrt{5x-9}\right)^2$	Square both sides to eliminate the square root.
$\left(x-1\right)\left(x-1\right)=5x-9$	Binomial Expansion
$x^2-2x+1=5x-9$	Multiplied Binomials
$x^2-7x+10=0$	Set equal to zero
$\left(x-5\right)\left(x-3\right)=0$	Factored
$x=5$ or 3	Used Zero Product Property
$5-1=\sqrt{5(5)-9}$ $\quad$ $3-1=\sqrt{5(3)-9}$ $4=\sqrt{16}$ $\qquad\qquad$ $2=\sqrt{4}$ $2\neq\sqrt{6}$ $\qquad\qquad$ $2=2$	Check for Extraneous Solutions!

Assessment:

Exit Ticket:
Match the reasons to the steps. Pick the step the step one would take to get to the next line in the equation. Reason could be used more than once and some not at all.

1) $2\sqrt{x-7} - 6 = 12$

       Given

2) $2\sqrt{x-7} = 18$

3) $\sqrt{x-7} = 9$

4) $x - 7 = 81$

5) $x = 88$

a) Take the square root

b) Division Property of Equality

c) Addition Property of Equality

d) Subtraction Property of Equality

e) Square both sides

Warm-Up:

CST/CAHSEE:	Review:
Which of the following conclusions is true about the statement below? $$x^2 = \sqrt{x}$$ **A** The statement is always true. **B** The statement is true when x is negative. **C** The statement is true when $x = 0$. **D** The statement is never true. What are some issues that arise when teaching students about a variable as a radicand?	Let $x = 3$ and $x = 5$. Determine if both numbers are solutions to the equation $$x - 1 = \sqrt{5x - 9}$$

Current:

Solve $2(3x + 5) + 6 = 10$ using two different methods. Describe the method.

Today's Objective/Standards: Common Core Standard: A-REI-2: Solve simple rational
and radical equations in one variable and give examples how extraneous solutions may arise.

Warm-Up: <u>Answer Key</u>

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Which of the following conclusions is true about the statement below? $$x^2 = \sqrt{x}$$ **A** The statement is always true. **B** The statement is true when x is negative. **Ⓒ** The statement is true when $x = 0$. **D** The statement is never true.	Let $x = 3$ and $x = 5$. Determine if both numbers are solutions to the equation $$x - 1 = \sqrt{5x - 9}$$ **Only** $x = 5$ **is a solution**.
What are some issues that arise when teaching students about a variable as a radicand?	

Current:

Solve $2(3x + 5) + 6 = 10$ using two different methods. Describe the method.

Samples:

$$\frac{2(3x+5)}{2} + \frac{6}{2} = \frac{10}{2}$$
$$(3x+5) + 3 = 5$$
$$(3x+5) = 2$$
$$3x = -3$$
$$x = -1$$

$$2(3x+5) + 6 = 10$$
$$2(3x+5) + 6 = 4 + 6$$
$$2(3x+5) = 4$$
$$2(3x+5) = 2 \cdot 2$$
$$(3x+5) = 2$$
$$3x = -3$$
$$x = -1w$$

Today's Objective/Standards: **Common Core Standard:** A-REI-2: Solve simple rational and radical equations in one variable and give examples how extraneous solutions may arise.